

Cosmic Resilience: Galactic Discs as a Universal Laboratory for Emergent Order in Complex Systems

Project Summary

Understanding how complex systems maintain stability in the face of constant external and internal perturbations is a fascinating challenge that transcends disciplinary boundaries. This project proposes to use the universe's largest and oldest natural experiments – galaxies – as a unique laboratory to study the mechanisms of resilience and self-organization. For billions of years, vast stellar discs, seemingly fragile structures, have demonstrated remarkable robustness, resisting disruption and maintaining a highly ordered state. This persistence cannot be explained by conventional models that rely on fine-tuned, localized parameters.

We posit that galactic discs are archetypal examples of emergent, homeostatic systems. Their resilience does not stem from a delicate balance but from a robust, self-regulating feedback loop driven by the universal force of gravity. In this framework, perturbations that "heat" the cool system by increasing orbital disorder trigger a powerful gravitational response, which in turn drives cooling processes (star formation on ordered orbits) that challenge stability. The system is naturally drawn to an attractor state of marginal stability, allowing it to absorb perturbations and persist over cosmic timescales. In turn this attractor closely links the global properties of the galaxy to its local star formation efficiency, as traced by various observed scaling laws.

The core methodological innovation of this project, COSMIC RESILIENCE, is to move beyond computationally intensive numerical simulations towards a predictive, analytical understanding relying on joint expertise from researchers from INSU and INSMI. We will develop and validate a novel "dressed" reaction-diffusion formalism derived from kinetic theory. This mathematical framework will provide a powerful tool to model the interplay between stochastic heating (diffusion) and deterministic cooling (reaction), mediated by the system's own collective response.

By testing this model against the unparalleled dataset provided by cosmological surveys, this project will not only transform our understanding of galaxy evolution but also aims to extract universal principles of self-organization and resilience applicable to other complex systems governed by long-range forces and feedback loops, from plasmas to biological or computational networks.

Keywords: Resilience, Robustness, Complex Systems, Self-Organization, Emergence, Homeostasis, Reaction-Diffusion Models, Kinetic Theory, Gravitational Dynamics.

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I Scientific Context and Position

Recent observations with the James Webb Space Telescope (JWST) reveal the widespread presence of thin galactic disks, including at high redshift, i.e. very early on in the age of the Universe [1]. This apparent structural robustness partly challenges our current understanding of structure formation within the standard cosmological model. While angular momentum conservation is often invoked to explain their stability, this explanation overlooks the fundamental role of multi-scale baryonic processes, which can establish a form of spontaneous gravitational self-regulation. This mechanism appears not only to stabilize disks against perturbations but also to reduce the scatter in observed scaling relations (Tully–Fisher, radial acceleration relation, etc.). Understanding the origin and persistence of these disks is essential for interpreting the morphological statistics that will be exploited by future large galaxy surveys (Euclid, LSST), and for improving the resulting cosmological constraints.

The MITI call for proposals highlights resilience as a key concept in understanding systems ranging from materials and ecosystems to computer science and societies. A central theme is the capacity of a system to resist perturbations and maintain its structural integrity and function. The present project, COSMIC RESILIENCE, addresses this theme directly by investigating one of the most dramatic and observable examples of long-term resilience in nature: the galactic disc.

A Fragile yet Robust Structure

Galactic discs are vast collections of stars and gas in nearly circular orbits, forming a dynamically "cold", thermodynamically improbable, and seemingly fragile structure [2]. According to naive models, these discs should be easily disrupted by gravitational perturbations from satellite galaxies, cosmic gas infall, or internal instabilities. Yet, observations show that thin, star-forming discs are ubiquitous across cosmic history, surviving for over ten billion years in a constantly changing environment [1]. Furthermore, their properties obey remarkably tight scaling laws, suggesting they are not random collections but are governed by a powerful ordering principle.

Challenging the Mainstream Paradigm

The dominant paradigm attempts to explain this stability with "bottom-up" feedback, where localized small-scale events like supernova explosions are finely tuned in simulations to regulate the entire galaxy [3]. This approach has significant limitations: it lacks predictive power, obscures causal links, and is too computationally prohibitive to explore wide parameter space.

We propose a paradigm shift towards a "top-down" understanding based on the principles of emergence and self-organization. We argue that the resilience of galactic discs is an intrinsic property of it being an open, dissipative system governed by the long-range force of gravity. These systems naturally find a homeostatic attractor state where their response to perturbations

robustly maintains their structure. This gravity-driven self-regulation provides a fundamental and predictive explanation for the observed order in the presence of thin galactic discs throughout the evolution of the Universe.

II Project Objectives

The primary goal of the present project is to formalize and validate a new physical theory for the resilience of intricate, self-gravitating systems. This will be achieved through the following objectives:

- **Develop a novel mathematical framework for resilience in the gravitational context.** We will formulate and solve a "dressed" reaction-diffusion equation derived from first-principles kinetic theory [4]. This master equation will describe the secular evolution of the disk's distribution function, capturing the balance between disorder-inducing perturbations (diffusion) and order-restoring feedback (reaction), mediated by the system's own long-range gravitational susceptibility.
- **Identify the universal parameters and thresholds that govern the cosmological robustness of galactic discs.** Using this analytical framework, we will explore the system's behavior under a wide range of conditions (e.g., varying the intensity and nature of external noise, the efficiency of the cooling term). Our goal is to identify the critical thresholds where self-regulation fails, leading to a catastrophic phase transition and morphological transformation (i.e., the "death" of the disc), e.g., evolving into an elliptical galaxy [5]. Doing so, we will quantitatively assess the extent of the resilience of galactic discs.
- **Validate the model against large-scale surveys.** The predictions of our framework – such as the tightness of scaling laws, the system's stability as a function of cosmic environment, and the epoch of spontaneous emergence – will be rigorously tested against observational data from major cosmic surveys (Euclid, LSST, DESI, JWST). We stress that this is a unique opportunity to validate the present theory of complex systems against high-precision, state-of-the-art astrophysical data.

Expected Results

While carrying out these objectives, our main deliverables will be:

- A unified theoretical framework to explain the emergence and resilience of thin galactic disks, grounded in the physics of complexity;
- A physical interpretation of the reduced scatter in galactic scaling relations as a natural consequence of this self-regulation;
- Predictions for key observables (disk thickness, bar or bulge fractions, scatter in star formation–metallicity–kinematics relations, Tully–Fisher, radial acceleration relation, Kennicutt–Schmidt relation, etc.), directly comparable with JWST, Euclid, Roman and LSST data;

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- Tools to marginalize morphology-induced biases in the inference of cosmological parameters.

III Methodology and Innovation

The Paris-Orleans collaboration, that will involve the “Institut d’Astrophysique de Paris” (IAP, INSU) and the “Institut Denis Poisson” (IDP, INSMI) will rely on two recent joint developments:

- The works of [6, 7, 4], obtained at IAP, validating the application of kinetic theory to stellar systems and the role of orbital diffusion in the secular evolution of disks;
- The results of [8, 9, 10], obtained at IDP, on large deviation theory applied to stellar systems, providing a coherent framework to capture morphological diversity beyond the ensemble average expectation.

With the support of the MITI program, the goal of COSMIC RESILIENCE is to now construct dissipative quasi-linear models able to track the evolution of self-gravitating disks subject to gas inflows and to feedback induced by turbulent star formation. These models will describe how a gravitational regulation loop maintains disks near marginal stability, with increasing efficiency over cosmic time. Large deviation theory will be the working horse to assess the efficiency of self regulation. This is precisely where the expertise on bifurcation theory [11] of IDP comes into play.

The methodological core of this project represents a rupture from the state-of-the-art in the field, which has relied almost exclusively on computationally expensive N -body simulations. While useful, such simulations are often difficult to interpret physically and understand the system’s fundamental behaviour. Our approach shifts the focus from mere simulations to analytical insight, providing explanatory and parametrised predictive power.

The "Dressed" Reaction-Diffusion Equation

The key methodological innovation is the application of kinetic theory to derive a master equation for the system’s long-term evolution in the form of a reaction-diffusion equation.

Indeed, the stellar component usually dominates the baryonic mass of galactic disks. Importantly, gravity is a long-range force, and typically the amplitude of perturbations is weak compared to the galaxy’s overall gravitational field. This means that the statistical impact of perturbations on galactic disks can be handled perturbatively. This is the key starting for COSMIC RESILIENCE that will derive, from kinetic theory, a generic "dressed" reaction-diffusion equation. This approach is a natural extension of established formalisms but introduces critical new components:

- **The Diffusion Term:** This term represents the physical process of orbital diffusion [12, 13, 14]. It models how stochastic gravitational perturbations – both internal (e.g., feedback driven density fluctuations) and external (e.g., cosmic infall, mergers) – act as a "heating" source, increasing the velocity

dispersion of the stars and, therefore, the structural disorder of the galactic disk [15].

- **The Reaction Term:** This is a novel addition that models the countervailing "cooling" processes. In the galactic context, this corresponds to the accretion of dynamically cold gas, from the cosmic large-scale structures, and the subsequent formation of new stars on highly ordered, near-circular orbits [16]. This term acts as a sink for dynamical energy, reducing disorder and driving the system back towards a stable state.
- **The "Dressing":** In this project, we will emphasise how the most crucial physical ingredient is the self-consistent "dressing" of the reaction-diffusion equation, by incorporating the collective gravitational response (the susceptibility) of the disc itself [17, 18]. Perturbations are not passive; they are quadratically amplified by the system’s own self-gravity in a process known as "swing amplification" [19, 20]. This dressing self-consistently accounts for how the system’s state mediates the interplay between heating (diffusion) and cooling (reaction), leading to the emergence of a robust homeostatic attractor state. More precisely, we argue that resilience is achieved through a rescaling of the effective secular time, solely permitted by the long-range nature of the gravitational interaction.

These key physical ingredients are illustrated in Fig. 1. At this stage we stress that, jointly, IAP and IDP have all the required scientific expertise to model in detail these various contributions that all contribute to the long-term resilience of galactic discs.

Once validated, this reaction-diffusion formalism will be a major break-through in a field that has almost exclusively relied on computationally expensive N -body Monte Carlo simulations to model gravity. The core strategy is to shift the narrative from merely simulating observations, to explaining why galactic evolution occurs, through analytical insights into self-regulation operating in a distinct regime.

The stationary solutions of the reaction-diffusion equation can be computed explicitly. Consequently, galaxy morphology or metallicity can be predicted conditional to a given environment or redshift, and incorporated into physically motivated bias models (e.g., in the context of the cosmic web, clusters, etc.). This improves the accuracy of standard rulers in cosmology. It also provides a physical model for galaxy bias. By incorporating this physically motivated understanding of how shape and orientation connects to mass and environment, it is for instance possible to generate more accurate corrections for systematic effects in weak lensing data [21]. This leads to more robust and precise cosmological constraints on the nature of dark energy.

Strategy and timeline

Our first goal will be to validate the dressed reaction diffusion framework via idealised models of increasing complexity, relying on the modularity of the perturbative framework: i) open/closed loop, ii) active/passive poten-

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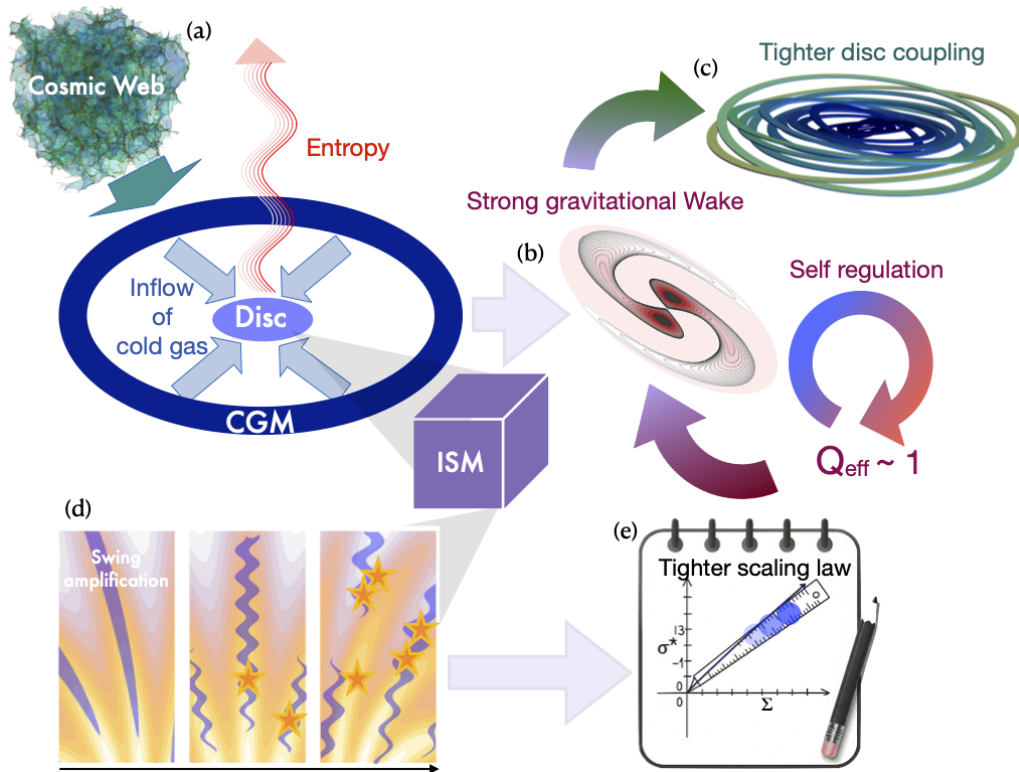


Figure 1. Illustration of the self-regulating control loop motivating the resilience and robustness of disc emergence. External and internal perturbations (heating/diffusion) are amplified by the system’s collective response, which in turn drives a powerful cooling (reaction) process, robustly maintaining the system in a state of homeostasis near marginal stability. The interplay between three co-evolving galactic components (a), the interstellar medium (purple), the disc (light blue) and the circum-galactic medium (dark blue), sets up an emerging dissipative machine (b), which, through wakes, achieves both self-regulation and stiffening, (c). Inflowing cold gas lowers the disc’s effective temperature, hence triggers wakes, which sources the turbulent cascade in the ISM through sequences of stellar swing amplifications, (d), leading to recurrent star formation. The thin disc inherits the coherence of the cosmic web’s steadiness, (a), through gravitationally-driven top-down causation, but the link is not finely tuned, thanks to the co-induced homeostasis towards stellar marginal stability. The net effect of the induced attractor is to “glue” baryonic and dynamical properties together, explaining the ubiquity of tight scaling laws, (e). Stellar or AGN feedback are not fine-tuned to reach bottom-up baryonic regulation, because it is achieved by the dominant stellar component.

tial [22], iii) with, w/o new stars, iv) with, w/o external noise [23], v) with different power spectra capturing a ranges of fluctuations (supernovae, giant molecular cloud, turbulence, etc.) [24], vi) with w/o gas, vii) assuming infinite/finite resolution, viii) as a local/global model, conditional to larger scale environment (cluster, group, cosmic web) [25], ix) or while varying the adiabatic parameters of the model (e.g., bulge/disc ratio). We stress that the track record of the IAP team is a clear testimony that it possesses all the needed astrophysical and dynamical expertise to carry out these challenging projects. The various stages of COSMIC RESILIENCE are illustrated in Fig. 2.

On the front of theoretical developments, we will focus on updating and generalising the formalism of large deviations to the case of galactic discs, progressing in steps of increasing difficulty: i) from dynamically hot, i.e. large velocity dispersion, to dynamically cold self-gravitating systems, i.e. galactic discs with low radial dispersion [26]; ii) from homogeneous, i.e. straight lines orbits to inhomogeneous systems, i.e. rosette orbits [27]; iii) from describing ensemble-averaged dynamics, i.e. via the Landau equation,

to intrinsically stochastic dynamics, i.e. via the Dean–Kawasaki equation [28, 9]; iv) from strongly stable, i.e. systems weakly amplifying perturbations, to marginally stable discs, i.e. one with (very) large linear amplification [29]. Benefiting from the expertise of the IDP team at Orleans, we will also investigate the use of “bifurcation theory” [30] to characterise the dynamics of fluctuations in self-gravitating systems close to “critical opalescence” [31], i.e. systems marginally stable. Finally, we also leverage the expertise of the IAP team at Paris to use techniques stemming from “renormalisation theory” [32] to characterise, a priori, the effective level of saturation in emergent galactic discs. For this part of COSMIC RESILIENCE we stress that the contribution from the IDP will be instrumental in ensuring that state-of-the-art mathematical techniques are efficiently used to truly kinetic theory at the service of quantitatively constraining galactic emergence, as illustrated by the recent track record of this team.

Finally, we will investigate the domain of efficiency of the self-regulation and contrast it to the standard paradigm. More precisely, we also investigate when self-regulation fails, quenching star formation and inducing disruptive

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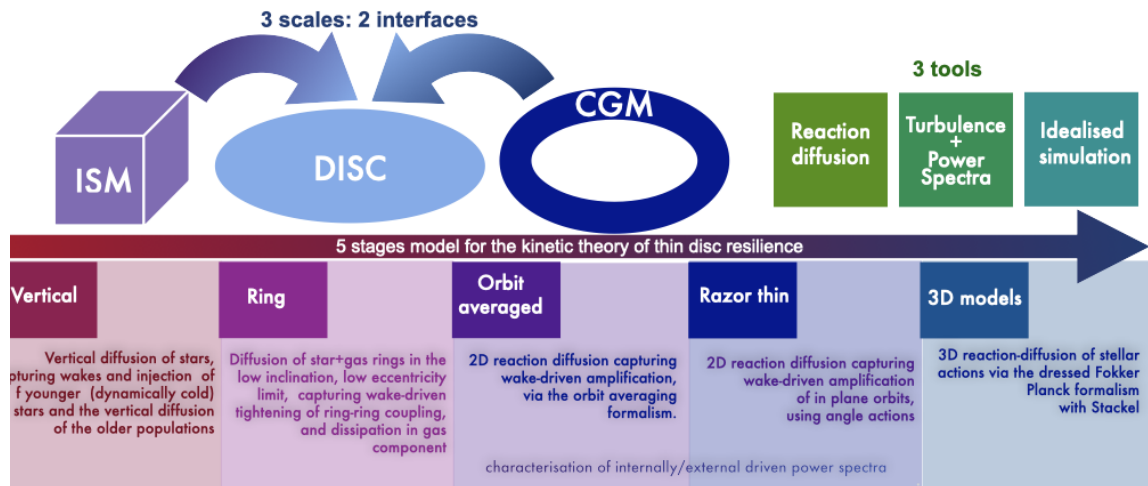


Figure 2. The various stages of modelling the emergence and resilience of stellar discs that will be addressed by COSMIC RESILIENCE in the context of the framework reaction diffusion and large deviation theory.

morphological transformation, i.e. transforming a disk galaxy into an elliptical one. Doing so, we will derive thresholds for unregulated or quenched galaxies, ellipticals, bulges and bars. We will explore the possibility that the two pathways to regulation may not be mutually exclusive but may describe different, complementary phases of a galaxy's life. The bottom-up narrative may explain the chaotic early formation of galaxies while the emergent explanation applies to the remarkably ordered nature of the mature, star-forming disks we observe later.

Practical expenses

Computing. The theoretical developments promoted by COSMIC RESILIENCE require dedicated hardware to perform extensive numerical validation using tailored N -body simulations. Our scientific track record is a clear testimony that they are in a position to fully leverage the capabilities of state-of-the-art computing architectures. In practice, we budget funds to access the local “infinity” computing cluster at IAP (<https://infinity-cluster.projet-horizon.fr/>). This cluster offers all the needed infrastructure for electricity, cooling, long-term storage and system administration. We will leverage already existing hardware, rather than investing in new one, to reduce the program's total carbon footprint. For the 2-year duration of COSMIC RESILIENCE, we budget for the continuous and dedicated access to two nodes with 128 cores (AMD EPYC 9534), each with 16x32GB of RAM. This amounts to a total of ~ 2.5 millions CPU-hours throughout the project. In this environment, COSMIC RESILIENCE will also have shared limited access to large storage (~ 10 PB), state-of-the-art GPUs (H100) and ~ 5000 CPU cores. Within the globally integrated environment of “infinity”, the dedicated access to one core is charged 0.05€ per day (128 cores per node). For the overall duration of this project, this leads to a requested budget of 9 000€, i.e., 4 500€ per year.

Mobility. Participation in workshops and conferences is paramount for disseminating COSMIC RESILIENCE's

scientific results, as well as for obtaining up-to-date information on the latest developments. We emphasise that this is all the more valuable for early-career researchers. Each PhD student (Thomas Bizien, Sabri Errachdi, both at IAP, and Anwar El Rihirhayi at IDP) will take part in one summer school to further build their scientific expertise, as well as two conferences per year: one national meeting or workshop, as well as one international one. As such, we request 2 000€, per year, per PhD student to support their mobility. We will also fund recurrent mobility between Paris and Orleans, and the opportunity to present our work in Toulouse, Marseilles, Lyon and Strasbourg. In order to build new collaborations and increase the international visibility of COSMIC RESILIENCE's results, for each of the two year of the program, we will organise three missions (for team members or to invite external collaborators), and request 2 000€ for each such mission. In practice, travels from France or nearby European countries will be made by train to mitigate the carbon footprint associated with COSMIC RESILIENCE. Throughout the duration of COSMIC RESILIENCE, we will organise two international workshop at Cargese (in Oct 26 and May 2027 respectively) involving M. Weinberg, S. Prunet, M. Petersen, B. Famaey, A. L. Varri, J. Magorrian, P.-H. Chavanis, S. de Rijcke. We will also organise smaller-scale national workshops in Paris and Orleans respectively for the PhD students and J.-B. Fouvry, C. Pichon, C. Cadiou, J. Barré. respectively. We request 8 000€ per year to cover the associated costs.

Grand-total Overall, this sums up to 25 500€ per year for the total costs.

IV Interdisciplinarity and Impact

Interdisciplinarity and New Synergies

This project is inherently interdisciplinary, creating synergies between teams (IAP – INSU and Université d'Orleans – INSMI) as well as between fields that rarely interact directly:

- **Statistical Physics and Kinetic Theory:** We ap-

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ply foundational concepts like the Balescu-Lenard equation and large deviation theory to self-gravitating systems.

- **Applied Mathematics:** The core of the project is the development and solution of a novel type of non-linear reaction-diffusion equations.
- **Computational Science:** High-performance computing is not just a tool for matching data and profiling new instruments but also a validation testbed for our analytical theories.
- **Astrophysics and Cosmology:** The project uses the cosmos as a laboratory and, in turn, provides physically motivated models to improve the analysis of major sky surveys.

The project team (Institut Denis Poisson and Institut d'Astrophysique de Paris) reflects this interdisciplinarity, bringing together experts in secular gravitational dynamics, kinetic theory, large deviation theory, interstellar medium turbulence, numerical simulations, and observational astronomy.

Exploratory Character and Risk Assessment

COSMIC RESILIENCE is an exploratory, high-risk, high-reward project.

- **The Risk:** The primary risk lies in the ambitious attempt to distill the multi-scale, non-linear dynamics of a galaxy into a single, albeit sophisticated, master equation. Galaxy formation is notoriously complex, and our analytical framework may not capture all relevant physics.
- **Mitigation:** We mitigate this risk through a staged approach, continually validating our analytical results against targeted, idealized numerical simulations at every step of increasing complexity. The project team has a proven track record of successfully pioneering this type of theoretical work.

The potential payoff is a paradigm shift in a major field of astrophysics, moving from empirical calibration to a gravity based, first-principles, predictive theory. More broadly, it is the development of a powerful new tool for understanding resilience in a wide class of complex systems.

Expected Impact and Valorization

The impact of this project will be twofold:

- **Disciplinary Impact:** COSMIC RESILIENCE will provide a definitive physical explanation for the remarkable resilience of galactic discs and the origin of their tight scaling laws. This will resolve a decades-old puzzle in astrophysics and provide essential, physically-motivated models to reduce systematic biases in flagship cosmological surveys like Euclid, thereby improving constraints on dark energy and the fundamental nature of the universe. When completed, COSMIC RESILIENCE will have demonstrated in detail how gravity coupled with baryons provides top-down causation, from the cosmic web, via the circumgalactic medium, down to wake-controlled turbulent star formation and feedback in the intra-galactic medium. It will explain

the appearance, and most importantly the resilience over cosmic time of such fragile galactic structures.

- **Transdisciplinary Impact:** The true long-term value for the broader scientific community lies in establishing a concrete, verifiable model for *emergence and resilience in a real-world physical system*. By demonstrating how a simple, universal law (gravity) gives rise to complex, self-regulating structures, COSMIC RESILIENCE will provide a unique laboratory for studying concepts like homeostasis and self-organized criticality in astrophysical systems. The developed reaction-diffusion formalism, once validated, could serve as a template for modeling other systems where long-range interactions and feedback loops are critical, such as in confined plasmas, geophysical systems, or even complex biological or socio-economic networks. The results will be disseminated through high-impact publications, open-source analytical tools, and presentations at both specialized and interdisciplinary conferences.

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